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Free convection heat transfer from arrays of vertically separated horizontal cylinders at low Rayleigh numbers

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Abstract—Steady state free convection heat transfer from horizontal isothermal cylinders in vertical arrays of two to eight, at low Rayleigh numbers, is studied, experimentally. Effects of Rayleigh number and cylinder to cylinder separation distance on the heat transfer behavior of the cylinders are investigated. Heat transfer from the bottom cylinder remains the same as that of a single cylinder. However, for other cylinders we may have either reduction or improvement of heat transfer which depends on their location in the array and the geometry of the array. Results show that there is an optimum separation distance for the best overall convection heat transfer of each array. A correlation is presented to calculate the array Nusselt number in terms of Rayleigh number, cylinder spacing to diameter ratio and number of cylinders in the array.

INTRODUCTION

ONE OF the heat transfer problems with considerable engineering applications is the two dimensional free convection from a horizontal cylinder. Applications are high voltage power transmission lines, solar collectors, electronic devices, nuclear safety systems, and refrigeration condensers to name a few. Different configurations may be considered for this problem. One of the cases is heat transfer from a single horizontal cylinder both in free and confined spaces which has been studied extensively. A few of these investigations are the works of Kuehn and Goldstein [1], and Badr [2] for heat transfer from a cylinder in free space, and of Sparrow and Pfeil [3], Karim *et al.* [4], and Sadeghipour and Kazemzadeh Hannani [5] for heat transfer from a cylinder confined between two vertical walls.

Another configuration which is particularly important in a variety of heating and refrigeration equipments is free convection from a horizontal cylinder in an array set. Because of interaction between the temperature and flow fields around the neighboring cylinders, the heat transfer coefficient for a cylinder in an array is generally different from a single cylinder. Although, for certain cylinders in the array we may see no noticeable change in the heat transfer coefficient relative to a single cylinder. The net effect of this interaction is usually either reduction or enhancement of the average heat transfer coefficient for a cylinder. The trend and relative strength of this effect for any cylinder varies with its location in the array and the geometry of the array which includes the number, arrangement, and separation distance of the cylinders.

One of the first investigations of free convection heat transfer from the cylinders in an array set was done by Eckert and Soehngen [6]. They performed experiments for three cylinders of diameter 22.3 mm with uniform surface temperature in vertical in line, and staggered arrangements. In the case of vertical in line arrangement and for the separation distances between cylinders which they used, it seems that heat transfer from the lower cylinder in the array remains the same as a single cylinder. However, heat transfer from the middle cylinder reduces and there is even more reduction for the upper cylinder. In the staggered arrangement, the upper and lower cylinders were vertically in line with the middle cylinder displaced horizontally. The new arrangement has improved heat transfer from the middle cylinder.

Experimental studies have been carried out for vertical and inclined flat arrays of the cylinders with uniform heat flux by Lieberman and Gebhart [7] who used ten wires of the diameter 0.127 mm and by Marsters [8] who used three, five and nine cylinders of diameter 6.35 mm.

Sparrow and Niethammer [9] studied free convection heat transfer from an array of two vertically separated horizontal cylinders of diameter 37.9 mm, in the range of Rayleigh numbers from 20 000 to 200 000. It was found that there was no noticeable change in heat transfer from the lower cylinder relative to a single cylinder. However, the cylinder-to-cylinder temperature difference and separation distance had a great effect on heat transfer from the upper cylinder. In the case of cylinders at the same temperature, heat transfer from the upper cylinder decreased at small separation distances and increased at large ones. This phenomenon was predictable, because the plume from the lower cylinder influences the heat transfer characteristics of the upper cylinder in two different ways. First, it behaves as a forced convection field for the

NOMENCLATURE

A	area
D	cylinder diameter
F_{12}	shape factor between two adjacent cylinders
$F_{1\infty}$	shape factor between a single cylinder and the ambient
h	convective heat transfer coefficient
K	thermal conductivity
N	number of cylinders in an array
Nu	Nusselt number
Pr	Prandtl number
Q	thermal energy
Ra	Rayleigh number
S	cylinder to cylinder separation distance
T	temperature
X	location of a cylinder in the array, measured from the center of the lowest cylinder.

Greek symbols

ε	emissivity
σ	Stephan–Boltzmann constant.

Subscripts

conv	heat transfer by convection
elec	heat generated by electrical power
o	single cylinder condition
rad	heat transfer by radiation
s	surface condition
∞	outside condition.

Superscript

$\bar{}$	array condition.

upper cylinder and, second, it causes a lower temperature difference between the cylinder's surface and the adjacent air. The first effect tends to increase and the second to decrease heat transfer from that cylinder. The relative strength of each of these effects finally determines the net effect of the lower cylinder on the heat transfer from the upper cylinder.

Tokura *et al.* [10] have considered free convection heat transfer from vertical arrays of two, three and five horizontal cylinders of the diameter 28.5 mm, maintained at a uniform temperature, in the range of Rayleigh numbers from 28 000 to 280 000 in free space and between two parallel confining walls. They have studied the effects of cylinder spacing, separation distance between the confining walls and number of cylinders, on the heat transfer behavior of each cylinder in the arrays. They have measured both local and average heat transfer coefficient. They have given recommendations for selection of the optimum cylinder spacing, wall separation distance, and number of cylinders to enhance the total heat transfer from an array.

Sparrow and Boessneck [11] have investigated free convection from an array of two horizontal cylinders of diameter 37.9 mm, in the range of Rayleigh numbers from 20 000 to 200 000. They have performed experiments for the array in both vertical and oblique positions. They have found that heat transfer from the upper cylinder in the inclined position has improved relative to the vertical position.

The large diameter of the cylinders used in the above experiments have resulted in very large Rayleigh numbers. However, the range of Rayleigh numbers, corresponding to such cases as the operating conditions of the household refrigerator condensers, are very much lower than those values. Hunter and Chato [12] have performed experiments to study free convection heat transfer from the array of two and three horizontal cylinders of diameter 5.2 mm with uniform surface temperature in the range of Rayleigh numbers from 140 to 550. They have considered two different arrangements for the cylinders in the arrays. The two cylinders of the first array were vertically separated with horizontal offset. Arrangement of the cylinders in the second array was similar to that of the first one except for the third cylinder which was symmetrically set at the level of the lower cylinder to form an isosceles triangle. They found that for these arrangements, there was an improvement in the rate of heat transfer from the top cylinder relative to that of a flat vertical array.

Farouk and Guceri [13] have investigated the laminar and turbulent free convection heat transfer from single, and double in line and staggered rows of isothermal cylinders, numerically. They have studied the effect of varying cylinder spacing in the horizontal direction on heat transfer from the cylinders.

In the present experimental investigation natural convection heat transfer from the flat vertical arrays of two to eight horizontal isothermal cylinders of diameter 6.6 mm, for Rayleigh numbers 500, 600 and 700, has been studied. Effects of the cylinder-to-cylinder separation distance, the number of cylinders and Rayleigh number on the Nusselt number for the individual cylinders and for the array sets has been determined.

EXPERIMENTAL APPARATUS AND MEASUREMENTS

Cylinders of diameter 6.6 mm and length 774 mm with electrical resistance wires inside are used as the heated cylinders. The 664 mm length from the middle part of each cylinder is heated and used as the test section. Experiments showed that the temperature distribution over the test section of the cylinders was relatively uniform. The unheated segments of the cylinders were insulated, and to minimize the end losses, two small caps of approximately 10 mm in length, were attached to the ends of each cylinder.

Cylinders were mounted on a wooden frame with adjustable cylinder to cylinder separation distance. As the experiments were performed for the low Rayleigh numbers, the results were very sensitive to any outside disturbances. Therefore, the entire set up was covered with a transparent plastic sheet on the sides and the top kept open. A distance was kept between the sheet and the floor to make sure of the free passage of air from outside into the covered section. The lowest cylinders of the arrays were located at such a height that there was no effect from the floor on their heat transfer. Figure 1 shows a schematic view of the cylinders in an array.

Heat transfer from the test section of the cylinders was determined by measuring the power input to the resistance wires. Wires were assumed to have constant resistance. Since electrical output of thermocouples is very low, its electrical signal cannot be used for the control purposes without appropriate amplification. Therefore, LM335 temperature sensors with 0.1°C precision were used to measure the surface temperatures of the cylinders at the center. A similar sensor was used to measure the ambient temperature.

The temperature measurements were made on the top of the cylinders, where the boundary layer is thickest. The cylinder surface temperature is expected to be the same at any angular position. This is justified because

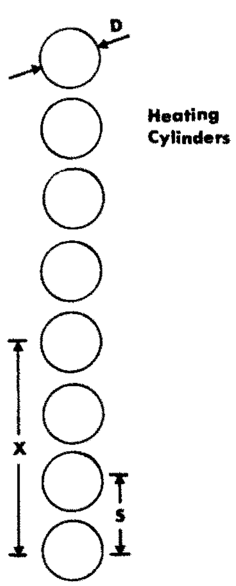

FIG. 1. A schematic view of the cylinders in an array.

the diameter of the heating rod is quite small and its thermal conductivity quite large [12]. In this experiment, we have kept uniformity of temperature between the cylinders within a tolerance of 0.2°C. This tolerance has been 1°C in the experiment by Tokura *et al.* [10], which is not acceptable for the case of our experiment at low Rayleigh numbers, because, it may cause about 10% error in Nusselt number. Our need for such a tolerance in difference between the temperature of the cylinders led us to design an electronic control device, Fig. 2. A reference voltage, proportional to a finite temperature, is produced by a non-inverting voltage follower operational amplifier, Op-Amp I. This type of Op-Amp has a gain of unity and can serve as a power amplifier to convert a high impedance input into a low impedance output. The output voltage which is more stable was used as a reference for Op-Amp II which is a non-inverting differential input. The voltage produced by the high precision sensor LM335 was compared with the reference voltage and the voltage difference controlled the output voltage imposed on the resistors until the desirable temperature was obtained. The potentiometer in the feed back system of Op-Amp II allowed a smooth approach of the system to steady-state condition.

DATA ANALYSIS

The purpose of this investigation was to determine the average Nusselt number for the individual cylinders and arrays as a function of Rayleigh number and separation distance between the cylinders. The electrical power supplied converts into heat in the cylinders. The heat generated dissipates from the cylinders through their surface either by convection or radiation. Despite installation of the end caps and the insulation of the unheated segments, we still have some end losses. Experiment showed that the heat loss from the ends constitute about 0.5% of the total power supplied. This is of the same order of magnitude as the uncertainty of the power input which falls within 0.2–0.5%. Therefore, we neglect the heat transfer from the ends and write:

$$Q_{elec} = Q_{conv} + Q_{rad} \quad (1)$$

where Q_{conv} and Q_{rad} are the heat losses through the surface of the cylinders, excluding their ends, by convection and radiation, respectively.

$$Q_{conv} = hA(T_s - T_\infty) \quad (2)$$

$$Q_{rad} = \varepsilon\sigma F_{1\infty} A(T_s^4 - T_\infty^4). \quad (3)$$

Nusselt number is defined as:

$$Nu = \frac{hD}{k} \quad (4)$$

where

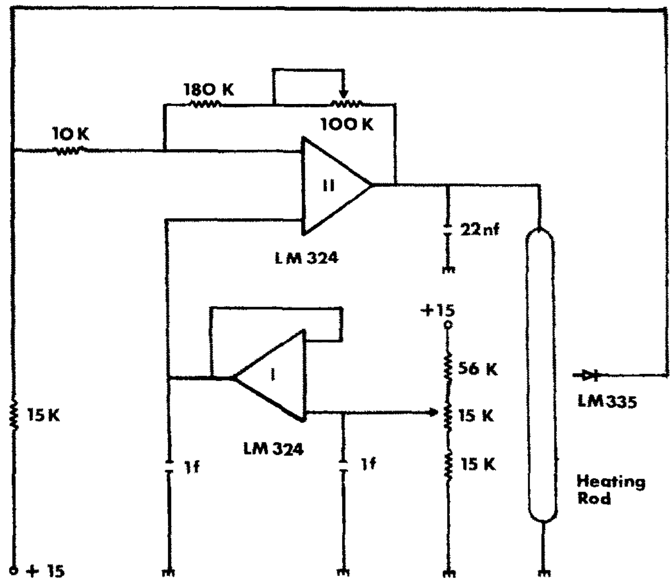

FIG. 2. The control system designed to sustain the uniformity of temperature between the cylinders.

$$h = \frac{Q_{elec}/A - \varepsilon\sigma F_{1,\infty}(T_s^4 - T_\infty^4)}{(T_s - T_\infty)}. \quad (5)$$

All thermodynamic properties of the air are evaluated at the film temperature.

Each cylinder loses heat by radiation either to the surrounding air or to its adjacent cylinders. Exchange of the thermal radiation between the cylinders can be neglected, due to the uniformity of temperature assumed for them. Depending on its location in the array, a cylinder has either one or two cylinders in its neighborhood. The shape factor between two long parallel cylinders is given as [14]:

$$F_{12} = \frac{2}{\pi}[\sin^{-1}(D/S) + ((S/D)^2 - 1)^{1/2} - (S/D)] \quad (6)$$

where S is the spacing between centers of the adjacent cylinders. Therefore, the shape factor between a cylinder and the ambient, $F_{1\infty}$, is given as:

$$F_{1\infty} = 1 - \alpha F_{12} \quad (7)$$

where, $\alpha = 1$, for the top and bottom cylinders and, $\alpha = 2$, for the others.

RESULTS AND DISCUSSION

A single horizontal cylinder

Correlations for the free convection heat transfer from a single horizontal cylinder in the open space are usually defined in the general form of

$$Nu = f(Ra, Pr). \quad (8)$$

For small variations in the temperature, Pr essentially remains constant and it can be dropped from equation (8). Correlations of this form are suggested by Tokura *et al.* [10] for $Ra = 28\,000$–$280\,000$, Morgan [15] for $Ra = 100$–$10\,000$, McAdams [16] for $Ra = 100$–1000, and Hunter and Chato [12] for $Ra = 150$–550. The Nusselt numbers for a single cylinder, obtained from our investigation for different Rayleigh numbers, are compared with those calculated from the correlations of [10, 12, 15, 16], in Fig. 3.

Before proceeding any further with our discussion, we need to discuss our basis for choosing a value for the surface emissivity of the cylinders, ε, used in equation (3). A sample of tabulated values, given in ref. [17], indicates that values of 0.85 up to 0.95 might be appropriate for ε. In the present work we chose $\varepsilon = 0.9$, which could cause a maximum 2.42% uncertainty in the values calculated for the Nusselt number. However, the perfect agreement of our Nusselt number with those obtained from the correlation of ref. [12], as compared in Fig. 3, suggests that our selection of $\varepsilon = 0.9$ is even closer to reality. Therefore, we may

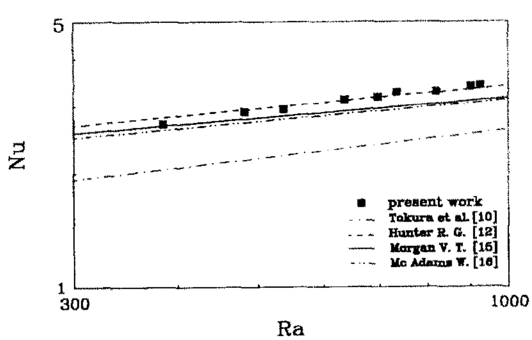

FIG. 3. Nusselt number vs Rayleigh number for a single cylinder in free space.

assume 2% uncertainty in the value chosen for ε, which corresponds to a maximum 1.21% uncertainty in the predicted values for the Nusselt number. Uncertainty in the measurement of Nusselt number, for a single cylinder, changes between 1.23 and 1.14% as Rayleigh number increases from 350 to 900.

A correlation of the form:

$$Nu_o = 0.62\ Ra^{0.25} \tag{9}$$

fits our data, with a maximum error of 1.9%.

Arrays of two to eight horizontal cylinders

Experiments are performed for air with $Pr = 0.72$, $Ra = 500, 600$ and 700, and $S/D = 3.5$ to 27.5 or 30.5, with $\Delta(S/D) = 3$. Our results show that, for any array with any number of cylinders, heat transfer from the lowest cylinder is the same as a single cylinder. Heat transfer from the upper cylinders could be either less or more than a single cylinder, depending on the geometry of the array, as was discussed before. For the small separation distances, higher temperature of the passing air will cause reduction in heat transfer from the upper cylinders. However, for the large distances the velocity effect becomes dominant which increases the heat transfer from the upper cylinders. The velocity (buoyant) effect for a cylinder in an array varies with the number of upstream and downstream cylinders.

Results for the upper cylinder in an array of two cylinders are presented in Fig. 4, in terms of their percent difference with those of a single cylinder. This term represents the percent enhancement or degradation of Nusselt number of the second cylinder relative to a single cylinder. Both degradation and enhancement of heat transfer from the upper cylinder can be explained based on the earlier discussions. Results from ref. [12] are also presented on the same figure which show a very good agreement with ours.

Average Nusselt number for any cylinder in an array of cylinders, excluding the lowest one, can be presented in the general functional form of:

$$Nu = a\ Ra^b (X/D)^c \tag{10}$$

where a, b and c are constants and X measures the location of a cylinder in the array relative to the center of the bottom cylinder. Application of a multivariable regression method to find the constants in equation (10) showed that b changes from 0.25 to 0.27 for different data sets. To be consistent with the results of ref. [10] we chose $b = 0.25$ for presentation of our results.

Variation of average Nusselt number, for the downstream cylinders in the arrays with two to eight cylinders, with Ra and X/D is illustrated in Figs. 5–11. It can be seen that the maximum value of average Nusselt number for the downstream cylinders in every array approaches an asymptotic value which varies

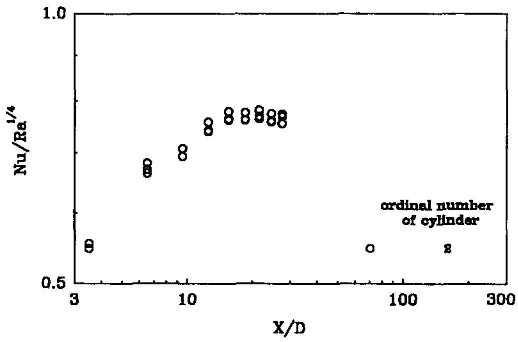

FIG. 5. Variation of the ratio $Nu/Ra^{1/4}$, for the upper cylinder in an array of two, with its location relative to the lower one.

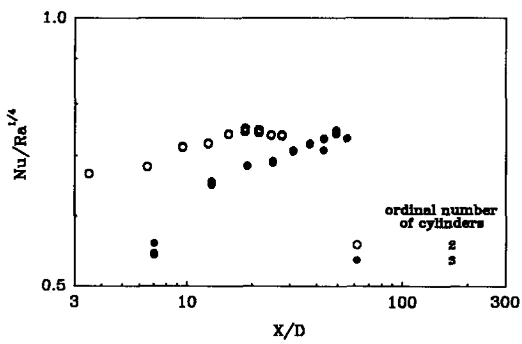

FIG. 6. Variation of the ratio $Nu/Ra^{1/4}$, for the cylinders in an array of three, with their locations relative to the lowest one.

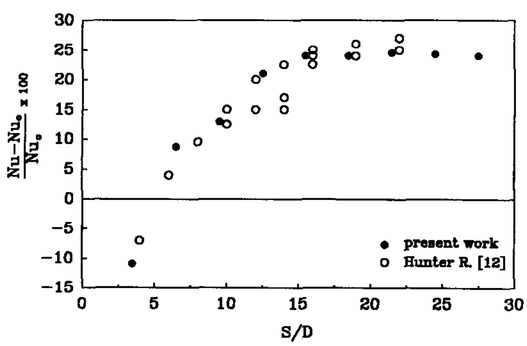

FIG. 4. Variation of Nusselt number for the upper cylinder in an array of two, relative to a single cylinder, with the cylinder spacing to diameter ratio.

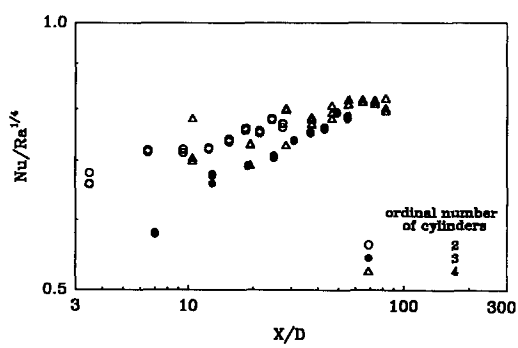

FIG. 7. Variation of the ratio $Nu/Ra^{1/4}$, for the cylinders in an array of four, with their locations relative to the lowest one.

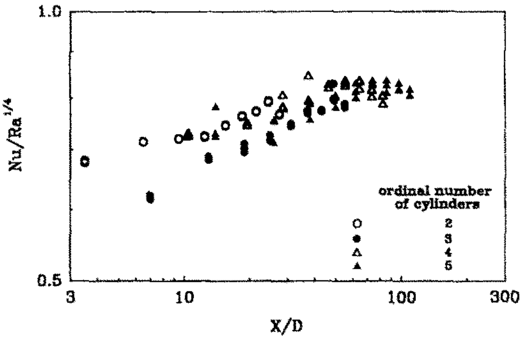

FIG. 8. Variation of the ratio $Nu/Ra^{1/4}$, for the cylinders in an array of five, with their locations relative to the lowest one.

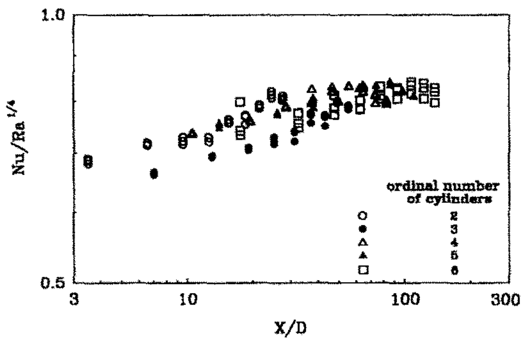

FIG. 9. Variation of the ratio $Nu/Ra^{1/4}$, for the cylinders in an array of six, with their locations relative to the lowest one.

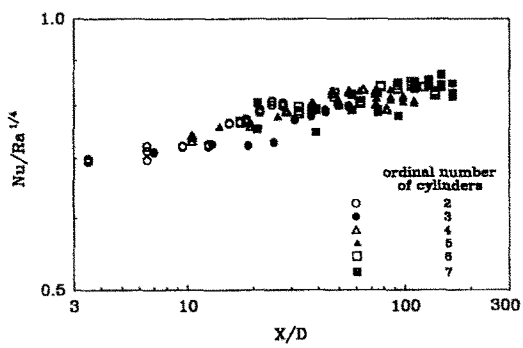

FIG. 10. Variation of the ratio $Nu/Ra^{1/4}$, for the cylinders in an array of seven, with their locations relative to the lowest one.

with the number of cylinders in the array. It increases with the number of cylinders in the array. This is due to the fact that the induced buoyant effect in an array with more cylinders generates a stronger flow field around the entire array. The mentioned effect also has a significant influence on the heat transfer behavior of the arrays in small separation distances and tends to oppose the effect of temperature increase in the vicinity of the downstream cylinders. Therefore, as the number of cylinders in an array increases the preliminary reduction in Nusselt number, at the small separation distances, gradually vanishes. Unlike the results from ref. [10], average Nusselt number of the

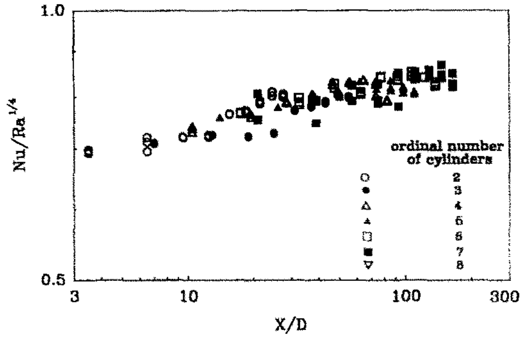

FIG. 11. Variation of the ratio $Nu/Ra^{1/4}$, for the cylinders in an array of eight, with their locations relative to the lowest one.

second cylinder in the arrays with more than two cylinders is higher than that of a two cylinder array.

Average Nusselt number for the arrays

One piece of information which is of great value for design purposes is the average Nusselt number for an array as a whole, $\overline{Nu}$. The array Nusselt number, $\overline{Nu}$, can be presented in the same functional form as was used for the individual cylinders, equation (10). Figure 12 shows variation of the ratio $\overline{Nu}/Ra^{1/4}$ for the arrays, with the cylinder spacing to diameter ratio, S/D. $\overline{Nu}/Ra^{1/4}$ is always higher for the arrays with more cylinders and this is because of their more powerful flow field relative to the arrays of less cylinders. Results show that, for $S/D > 15$, heat transfer from the arrays cannot be improved by increasing the separation distance. Our results behave similar to those of ref. [10] for $Ra = 28\,000$ to $280\,000$, however, with an approximate 30% increase in the ratio of $\overline{Nu}/Ra^{1/4}$. This was not unexpected, because their correlation for a single cylinder, for high Rayleigh numbers, also predicts $Nu/Ra^{1/4}$ about the same percent lower than ours, for low Rayleigh numbers.

Figure 13 is a different presentation of the results in Fig. 12 in which, the curves for different arrays are condensed into a single one. This curve depicts

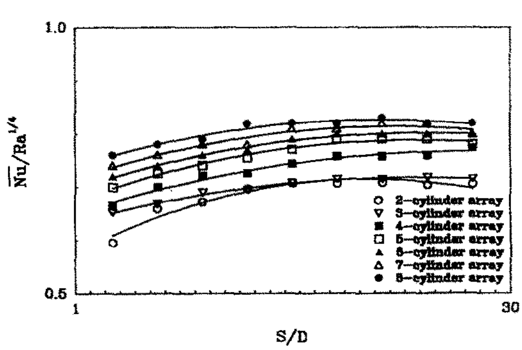

FIG. 12. The ratio $\overline{Nu}/Ra^{1/4}$ as a function of cylinder spacing to diameter ratio, S/D, for the arrays of two to eight cylinders.

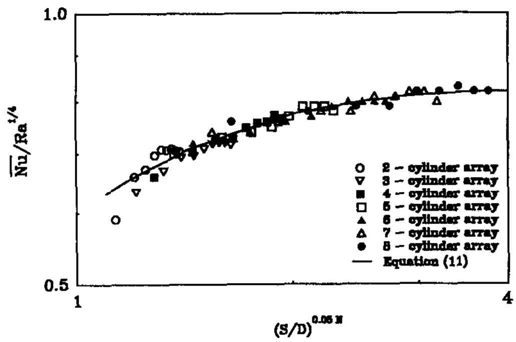

FIG. 13. A plot of the correlation (11), applicable for the arrays of two to eight cylinders, Rayleigh numbers from 500 to 700, and cylinder spacing to diameter ratio from 3.5 to 27.5.

variation of the ratio $\overline{Nu}/Ra^{1/4}$ with $(S/D)^{0.05N}$, where N is the number of cylinders in the array. Therefore, it can be used to predict Nusselt number for any array with any number of cylinders, in the range of our experiments. The function which represents the curve of Fig. 13 is:

$$\overline{Nu_N} = Ra^{1/4}[0.823+\exp(-1.5(S/D)^{0.05N})] \quad (11)$$

$$500 \leqslant Ra \leqslant 700, \quad 3.5 \leqslant S/D \leqslant 27.5, \quad 2 \leqslant N \leqslant 8.$$

This equation fits our data within a maximum error of 4%, however, for single value of $(S/D)^{0.05N} = 1.13$, the error increases to 7.4%.

CONCLUSIONS

In an array of the horizontal isothermal cylinders with any number of cylinders, the heat transfer coefficient for the lowest cylinder is the same as a single horizontal cylinder. However, for the downstream cylinders, heat transfer usually degrades for small separation distances and then improves relative to a single cylinder and approaches a constant value, as the separation distance increases. The degree of heat transfer degradation at small separation distances is less for the cylinders in smaller arrays. For the second cylinder in an array of two cylinders the maximum improvement in Nusselt number is about 24% which is attained for $S/D > 20$. For the arrays with more cylinders, the downstream cylinders obtain their maximum improvement for $S/D > 15$. However, the value of improvement is slightly higher for the arrays with more cylinders.

The array Nusselt number increases as the number of cylinders in the array increases. A correlation is presented, equation (11), which gives the array Nusselt number as a function of Ra, S/D and N.

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